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## III. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Suppose  $ABCD$  the required trapezoid,  $AB$  being the shorter (upper) base,  $BD=x$ , one diagonal,  $AC=BF=y$ ,  $BG$ =altitude= $h$ . Produce  $DC$  to  $F$  so that  $DF=DG+GF=DC+CF=a$ ,  $x+y=c$ ,  $\theta$ =given angle.  $\sqrt{(x^2-h^2)}+\sqrt{(y^2-h^2)}=a$ .  $4x^2y^2=a^4+(x^2+y^2)^2+4a^2h^2-2a^2(x^2+y^2)$ . But  $x^2+y^2=c^2-2xy$ .  $\therefore 4(c^2-a^2)xy=4a^2h^2+(c^2-a^2)^2$ . Now  $xy\sin\theta=ah$ .

$$\therefore 4a^2h^2-4ah\left(\frac{c^2-a^2}{\sin\theta}\right)=-(c^2-a^2)^2.$$

$$\therefore h=\frac{1}{2a}(c^2-a^2)\tan\frac{1}{2}\theta \text{ or } h=\frac{1}{2}(c^2-a^2)\cot\frac{1}{2}\theta.$$

$$\therefore xy=\frac{c^2-a^2}{4\cos^2\frac{1}{2}\theta} \text{ or } \frac{c^2-a^2}{\sin^2\frac{1}{2}\theta} \text{ and } x+y=c.$$

$$\therefore x=\frac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \frac{1}{2}c\pm\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta};$$

$$y=\frac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\cos^2\frac{1}{2}\theta)}}{2\sin\frac{1}{2}\theta} \text{ or } \frac{1}{2}c\mp\frac{\sqrt{(a^2-c^2\sin^2\frac{1}{2}\theta)}}{2\cos\frac{1}{2}\theta}.$$

Hence lay off  $DF=a$ , and draw  $AE$  parallel to  $DF$  at a distance  $h$  from it. With  $D$  as center and  $DB=x$  or  $y$  draw  $DB$ ; then  $DF=y$  or  $x$ . Draw any line parallel to  $BF$  as  $AC, A'C', A''C''$ , and join  $B, D$  to  $C, C', C'', A, A', A''$ , respectively. Then any one of the many trapezoids thus formed fulfill the required conditions, as is evident by drawing a figure.

## IV. Solution by J. J. KEYES, Nashville, Tenn.

Construct the triangle  $EBF$  having  $BE$ =sum of diagonals,  $BF$ =sum of bases, and angle  $E=\frac{1}{2}$  given angle. At  $F$  construct the angle  $EFD$ =angle  $E$ ,  $FD$  meeting  $BE$  in  $D$ . Through  $D$  draw  $DM$  parallel to  $FB$ . Take any point  $C$  on  $BF$ , draw  $CA$  parallel to  $FD$  meeting  $DM$  in  $A$ . Join  $AB, DC$ .  $ABCD$  is the required trapezoid. In proof,  $AO=CF$ .  $\therefore AD+BC=BC+BD$ =sum of bases.  $AC=DF=DE$ .  $\therefore AC+BD=BD+DE$ =sum of diagonals. The angle between  $BD$  and  $AC$ =angle  $BDF=2$  angle  $E$ =given angle.

Also solved by Elmer Schuyler.

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 CALCULUS.
 

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Problem number 181 was also solved by S. A. Corey and L. C. Walker.

183. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

$$\text{Evaluate } \int_0^{\infty} \frac{\sin 2nx dx}{(a^2+x^2)\sin x}.$$

Solution by G. B. M. ZERR, A. M., Ph. D.

Let  $u = \int_0^\infty \frac{\sin 2nx}{(a^2 + x^2) \sin x} dx$ , then

$$\frac{du}{dn} = \int_0^\infty \frac{2x \cos nx dx}{(a^2 - x^2) \sin x}, \quad \frac{d^2 u}{dn^2} = - \int_0^\infty \frac{4x^2 \sin 2nx dx}{(a^2 + x^2) \sin x} = 4a^2 u - 4 \int_0^\infty \frac{\sin 2nx dx}{\sin x}.$$

$$\text{Let } v = 8 \int_0^\infty \frac{\sin nx \cos nx dx}{\sin x} = 8 \int_0^\infty \frac{e^{n\pi y/2} - e^{-n\pi y/2}}{e^{2\pi y/2} - e^{-2\pi y/2}} \cos nx dx.$$

Let  $x\sqrt{-1} = \pi y$ ,  $dx = -\pi\sqrt{-1}dy$ ;

$$\begin{aligned} \therefore v &= -8\pi\sqrt{-1} \int_0^\infty \frac{e^{\pi ny} - e^{-\pi ny}}{e^{\pi y} - e^{-\pi y}} \cos(\pi ny\sqrt{-1}) dy \\ &= -8\pi\sqrt{-1} \left( \frac{\sin \pi n}{e^{\pi y\sqrt{-1}} + 2\cos \pi n + e^{-\pi y\sqrt{-1}}} \right) = -2\pi\sqrt{-1} \tan \pi n. \end{aligned}$$

$$\therefore v = 0, \text{ since } n = \text{any integer.} \quad \therefore \frac{d^2 u}{dn^2} - 4a^2 u = 0.$$

$$u = A e^{2an} + B e^{-2an}. \quad \text{When } n = 0, u = 0, \text{ and } B = -A. \quad \therefore u = A(e^{2an} - e^{-2an}).$$

$$\text{When } n = 1, u = \frac{\pi e^{-a}}{a} \text{ and } A = \frac{\pi e^{-a}}{a(e^{2a} - e^{-2a})}.$$

$$\therefore u = \frac{\pi e^{-a}}{a} \cdot \frac{e^{2an} - e^{-2an}}{e^{2a} - e^{-2a}} = \frac{\pi e^{-a}}{a} \cdot \frac{\sinh 2an}{\sinh 2a}.$$

184. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

If  $u = f(x, y)$ ;  $\xi = e^x y$ ;  $\eta = e^x$ ; show that

$$\frac{d^2 u}{dx^2} - y^2 \frac{d^2 u}{dy^2} - y \frac{du}{dy} = 4 \xi \eta \frac{d^2 u}{d\xi d\eta}.$$

Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

We have  $x = \frac{1}{2} \log \xi + \frac{1}{2} \log \eta$ ,  $y = \sqrt{\xi/\eta}$ , and therefore

$$\frac{\partial}{\partial \xi} = \frac{\partial x}{\partial \xi} \frac{\partial}{\partial x} + \frac{\partial y}{\partial \xi} \frac{\partial}{\partial y} = \frac{1}{2} \left( \frac{1}{e^x y} \frac{\partial}{\partial x} + \frac{1}{e^x} \frac{\partial}{\partial y} \right), \text{ whence } 2\xi \frac{\partial}{\partial \xi} = \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}.$$

In a similar way we get

$$2\eta \frac{\partial}{\partial \eta} = \frac{\partial}{\partial x} = y \frac{\partial}{\partial y}.$$